$$\rho = (0.138 \text{ u/fm}^3)(1.66 \times 10^{-27} \text{ kg/u})\left(\frac{1 \text{ fm}}{10^{-15} \text{ m}}\right) = 2.3 \times 10^{17} \text{ kg/m}^3.$$

Significance

a. The radius of the Fe-56 nucleus is found to be approximately 5 fm, so its diameter is about 10 fm, or 10^{-14} m. In previous discussions of Rutherford's scattering experiments, a light nucleus was estimated to be 10^{-15} m in diameter. Therefore, the result shown for a mid-sized nucleus is reasonable.

b. The density found here may seem incredible. However, it is consistent with earlier comments about the nucleus containing nearly all of the mass of the atom in a tiny region of space. One cubic meter of nuclear matter has the same mass as a cube of water 61 km on each side.

10.1 Check Your Understanding Nucleus X is two times larger than nucleus Y. What is the ratio of their atomic masses?

10.2 | Nuclear Binding Energy

Learning Objectives

By the end of this section, you will be able to:

- Calculate the mass defect and binding energy for a wide range of nuclei
- Use a graph of binding energy per nucleon (BEN) versus mass number (A) graph to assess the relative stability of a nucleus
- Compare the binding energy of a nucleon in a nucleus to the ionization energy of an electron in an atom

The forces that bind nucleons together in an atomic nucleus are much greater than those that bind an electron to an atom through electrostatic attraction. This is evident by the relative sizes of the atomic nucleus and the atom $(10^{-15}$ and 10^{-10} m, respectively). The energy required to pry a nucleon from the nucleus is therefore much larger than that required to remove (or ionize) an electron in an atom. In general, all nuclear changes involve large amounts of energy per particle undergoing the reaction. This has numerous practical applications.

Mass Defect

According to nuclear particle experiments, the total mass of a nucleus (m_{nuc}) is *less* than the sum of the masses of its constituent nucleons (protons and neutrons). The mass difference, or **mass defect**, is given by

$$\Delta m = Zm_p + (A - Z)m_n - m_{\text{nuc}} \tag{10.4}$$

where Zm_p is the total mass of the protons, $(A - Z)m_n$ is the total mass of the neutrons, and m_{nuc} is the mass of the nucleus. According to Einstein's special theory of relativity, mass is a measure of the total energy of a system ($E = mc^2$). Thus, the total energy of a nucleus is less than the sum of the energies of its constituent nucleons. The formation of a nucleus from a system of isolated protons and neutrons is therefore an exothermic reaction—meaning that it releases energy. The energy emitted, or radiated, in this process is $(\Delta m)c^2$.

Now imagine this process occurs in reverse. Instead of forming a nucleus, energy is put into the system to break apart the nucleus (Figure 10.6). The amount of energy required is called the total **binding energy (BE)**, E_{b}.

Binding Energy

The binding energy is equal to the amount of energy released in forming the nucleus, and is therefore given by

$$E_b = (\Delta m)c^2. \tag{10.5}$$

Experimental results indicate that the binding energy for a nucleus with mass number $A > 8$ is roughly proportional to the total number of nucleons in the nucleus, *A*. The binding energy of a magnesium nucleus (^{24}Mg), for example, is approximately two times greater than for the carbon nucleus (^{12}C).

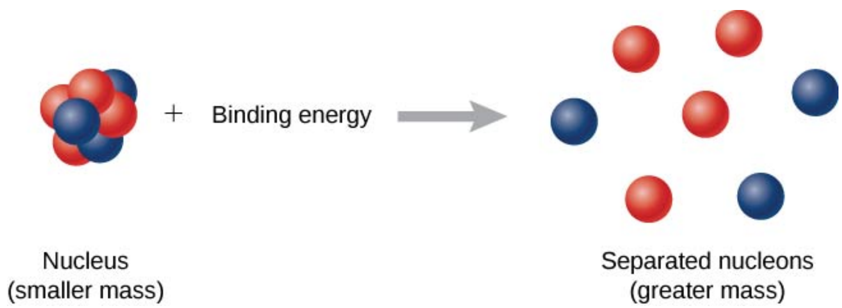

Figure 10.6 The binding energy is the energy required to break a nucleus into its constituent protons and neutrons. A system of separated nucleons has a greater mass than a system of bound nucleons.

Example 10.2

Mass Defect and Binding Energy of the Deuteron

Calculate the mass defect and the binding energy of the deuteron. The mass of the deuteron is $m_D = 3.34359 \times 10^{-27}\,\text{kg}$ or $1875.61\ \text{MeV}/c^2$.

Solution

From **Equation 10.4**, the mass defect for the deuteron is

$$\begin{aligned}\Delta m &= m_p + m_n - m_D \\ &= 938.28\ \text{MeV}/c^2 + 939.57\ \text{MeV}/c^2 - 1875.61\ \text{MeV}/c^2 \\ &= 2.24\ \text{MeV}/c^2.\end{aligned}$$

The binding energy of the deuteron is then

$$E_b = (\Delta m)c^2 = \left(2.24\ \text{MeV}/c^2\right)\left(c^2\right) = 2.24\ \text{MeV}.$$

Over two million electron volts are needed to break apart a deuteron into a proton and a neutron. This very large value indicates the great strength of the nuclear force. By comparison, the greatest amount of energy required to liberate an electron bound to a hydrogen atom by an attractive Coulomb force (an electromagnetic force) is about 10 eV.

Graph of Binding Energy per Nucleon

In nuclear physics, one of the most important experimental quantities is the **binding energy per nucleon (BEN)**, which is defined by

$$BEN = \frac{E_b}{A} \tag{10.6}$$

This quantity is the average energy required to remove an individual nucleon from a nucleus—analogous to the ionization energy of an electron in an atom. If the BEN is relatively large, the nucleus is relatively stable. BEN values are estimated from nuclear scattering experiments.

A graph of binding energy per nucleon versus atomic number *A* is given in **Figure 10.7**. This graph is considered by many physicists to be one of the most important graphs in physics. Two notes are in order. First, typical BEN values range from 6–10 MeV, with an average value of about 8 MeV. In other words, it takes several million electron volts to pry a nucleon from a typical nucleus, as compared to just 13.6 eV to ionize an electron in the ground state of hydrogen. This is why nuclear force is referred to as the "strong" nuclear force.

Second, the graph rises at low *A*, peaks very near iron $(\text{Fe},\ A = 56)$, and then tapers off at high *A*. The peak value suggests that the iron nucleus is the most stable nucleus in nature (it is also why nuclear fusion in the cores of stars ends with Fe). The reason the graph rises and tapers off has to do with competing forces in the nucleus. At low values of *A*, attractive nuclear forces between nucleons dominate over repulsive electrostatic forces between protons. But at high values of *A*, repulsive electrostatic forces between forces begin to dominate, and these forces tend to break apart the nucleus rather than hold it together.

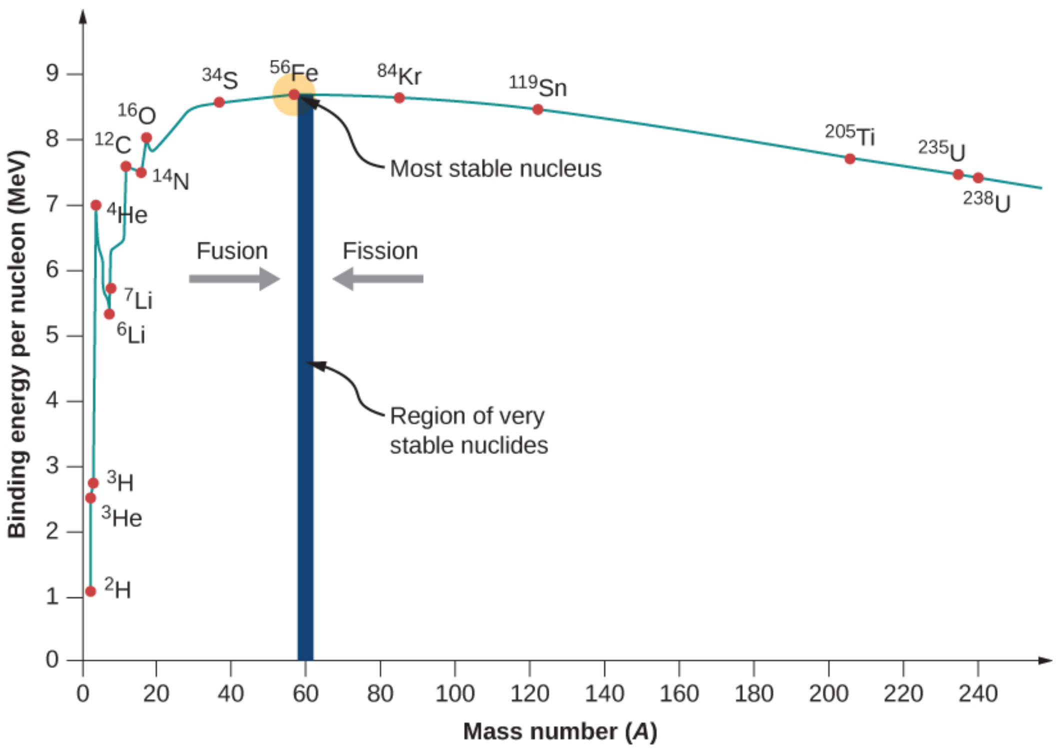

Figure 10.7 In this graph of binding energy per nucleon for stable nuclei, the BEN is greatest for nuclei with a mass near ^{56}Fe. Therefore, fusion of nuclei with mass numbers much less than that of Fe, and fission of nuclei with mass numbers greater than that of Fe, are exothermic processes.

As we will see, the BEN-versus-*A* graph implies that nuclei divided or combined release an enormous amount of energy. This is the basis for a wide range of phenomena, from the production of electricity at a nuclear power plant to sunlight.

Example 10.3

Tightly Bound Alpha Nuclides

Calculate the binding energy per nucleon of an $^{4}\text{He}\,(\alpha\text{ particle})$.

Strategy

Determine the total binding energy (BE) using the equation $\text{BE} = (\Delta m)c^2$, where Δm is the mass defect. The binding energy per nucleon (BEN) is BE divided by *A*.

Solution

For ^4He, we have $Z = N = 2$. The total binding energy is

$$\text{BE} = \{[2m_p + 2m_n] - m(^4\text{He})\}c^2.$$

These masses are $m(^4\text{He}) = 4.002602\text{ u}$, $m_p = 1.007825\text{ u}$, and $m_n = 1.008665\text{ u}$. Thus we have,

$$\text{BE} = (0.030378\text{ u})c^2.$$

Noting that $1\text{ u} = 931.5\text{ MeV}/c^2$, we find

$$\begin{aligned}\text{BE} &= (0.030378)(931.5\text{ MeV}/c^2)c^2 \\ &= 28.3\text{ MeV}.\end{aligned}$$

Since $A = 4$, the total binding energy per nucleon is

$$\text{BEN} = 7.07\text{ MeV/nucleon}.$$

Significance

Notice that the binding energy per nucleon for ^4He is much greater than for the hydrogen isotopes $(\text{only} \approx 3\text{ MeV/nucleon})$. Therefore, helium nuclei cannot break down hydrogen isotopes without energy being put into the system.

10.2 Check Your Understanding If the binding energy per nucleon is large, does this make it harder or easier to strip off a nucleon from a nucleus?

10.3 | Radioactive Decay

Learning Objectives

By the end of this section, you will be able to:

- Describe the decay of a radioactive substance in terms of its decay constant and half-life
- Use the radioactive decay law to estimate the age of a substance
- Explain the natural processes that allow the dating of living tissue using ^{14}C

In 1896, Antoine Becquerel discovered that a uranium-rich rock emits invisible rays that can darken a photographic plate in an enclosed container. Scientists offer three arguments for the nuclear origin of these rays. First, the effects of the radiation do not vary with chemical state; that is, whether the emitting material is in the form of an element or compound. Second, the radiation does not vary with changes in temperature or pressure—both factors that in sufficient degree can affect electrons in an atom. Third, the very large energy of the invisible rays (up to hundreds of eV) is not consistent with atomic electron transitions (only a few eV). Today, this radiation is explained by the conversion of mass into energy deep within the nucleus of an atom. The spontaneous emission of radiation from nuclei is called nuclear **radioactivity** (Figure 10.8).